Compound Interest

Lecture 3
Section 10.3

Robb T. Koether

Hampden-Sydney College

Fri, Aug 31, 2018

Outline

Compound Interest

Definition (Compound Interest)

When a loan is based on compound interest, interest is paid on the principal *and on all interest accrued so far*.

Definition (Compounding Period)

The compounding period is the length of time over which the interest is computed when it is compounded.

- The compounding period is usually expressed as the number of such periods per year.

Compound Interest

Example (Compound Interest)

- A principal of $1000 is invested at 5% interest compounded annually for 4 years.
- Find the future value.

Compound Interest

Example (Compound Interest)

- A principal of $1000 is invested at 5% interest compounded annually for 4 years.
- Find the future value.

Period	Principal	Interest	Balance
1	1000.00	50.00	1050.00

Compound Interest

Example (Compound Interest)

- A principal of $1000 is invested at 5% interest compounded annually for 4 years.
- Find the future value.

Period	Principal	Interest	Balance
1	1000.00	50.00	1050.00
2	1050.00	52.50	1102.50

Compound Interest

Example (Compound Interest)

- A principal of $1000 is invested at 5% interest compounded annually for 4 years.
- Find the future value.

Period	Principal	Interest	Balance
1	1000.00	50.00	1050.00
2	1050.00	52.50	1102.50
3	1102.50	55.13	1157.63

Compound Interest

Example (Compound Interest)

- A principal of $1000 is invested at 5% interest compounded annually for 4 years.
- Find the future value.

Period	Principal	Interest	Balance
1	1000.00	50.00	1050.00
2	1050.00	52.50	1102.50
3	1102.50	55.13	1157.63
4	1157.63	57.88	1215.51

Compound Interest

Example (Compound Interest)

- A principal of \$1000 is invested at 5% interest compounded semi-annually for 4 years.
- Find the future value.

Compound Interest

Example (Compound Interest)

- A principal of $1000 is invested at 5% interest compounded semi-annually for 4 years.
- Find the future value.

Period	Principal	Interest	Balance
1	1000.00	25.00	1025.00

Compound Interest

Example (Compound Interest)

- A principal of $1000 is invested at 5% interest compounded semi-annually for 4 years.
- Find the future value.

Period	Principal	Interest	Balance
1	1000.00	25.00	1025.00
2	1025.00	25.63	1050.63

Compound Interest

Example (Compound Interest)

- A principal of $1000 is invested at 5% interest compounded semi-annually for 4 years.
- Find the future value.

Period	Principal	Interest	Balance
1	1000.00	25.00	1025.00
2	1025.00	25.63	1050.63
3	1050.63	26.27	1076.90

Compound Interest

Example (Compound Interest)

- A principal of \$1000 is invested at 5% interest compounded semi-annually for 4 years.
- Find the future value.

Period	Principal	Interest	Balance
1	1000.00	25.00	1025.00
2	1025.00	25.63	1050.63
3	1050.63	26.27	1076.90
4	1076.90	26.92	1103.82

Compound Interest

Example (Compound Interest)

- A principal of $1000 is invested at 5% interest compounded semi-annually for 4 years.
- Find the future value.

Period	Principal	Interest	Balance
1	1000.00	25.00	1025.00
2	1025.00	25.63	1050.63
3	1050.63	26.27	1076.90
4	1076.90	26.92	1103.82
5	1103.82	27.60	1131.42

Compound Interest

Example (Compound Interest)

- A principal of $1000 is invested at 5% interest compounded semi-annually for 4 years.
- Find the future value.

Period	Principal	Interest	Balance
1	1000.00	25.00	1025.00
2	1025.00	25.63	1050.63
3	1050.63	26.27	1076.90
4	1076.90	26.92	1103.82
5	1103.82	27.60	1131.42
6	1131.42	28.29	1159.71

Compound Interest

Example (Compound Interest)

- A principal of $1000 is invested at 5% interest compounded semi-annually for 4 years.
- Find the future value.

Period	Principal	Interest	Balance
1	1000.00	25.00	1025.00
2	1025.00	25.63	1050.63
3	1050.63	26.27	1076.90
4	1076.90	26.92	1103.82
5	1103.82	27.60	1131.42
6	1131.42	28.29	1159.71
7	1159.71	28.99	1188.70

Compound Interest

Example (Compound Interest)

- A principal of $1000 is invested at 5% interest compounded semi-annually for 4 years.
- Find the future value.

Period	Principal	Interest	Balance
1	1000.00	25.00	1025.00
2	1025.00	25.63	1050.63
3	1050.63	26.27	1076.90
4	1076.90	26.92	1103.82
5	1103.82	27.60	1131.42
6	1131.42	28.29	1159.71
7	1159.71	28.99	1188.70
8	1188.70	29.72	1218.42

Outline

The Rule of 72

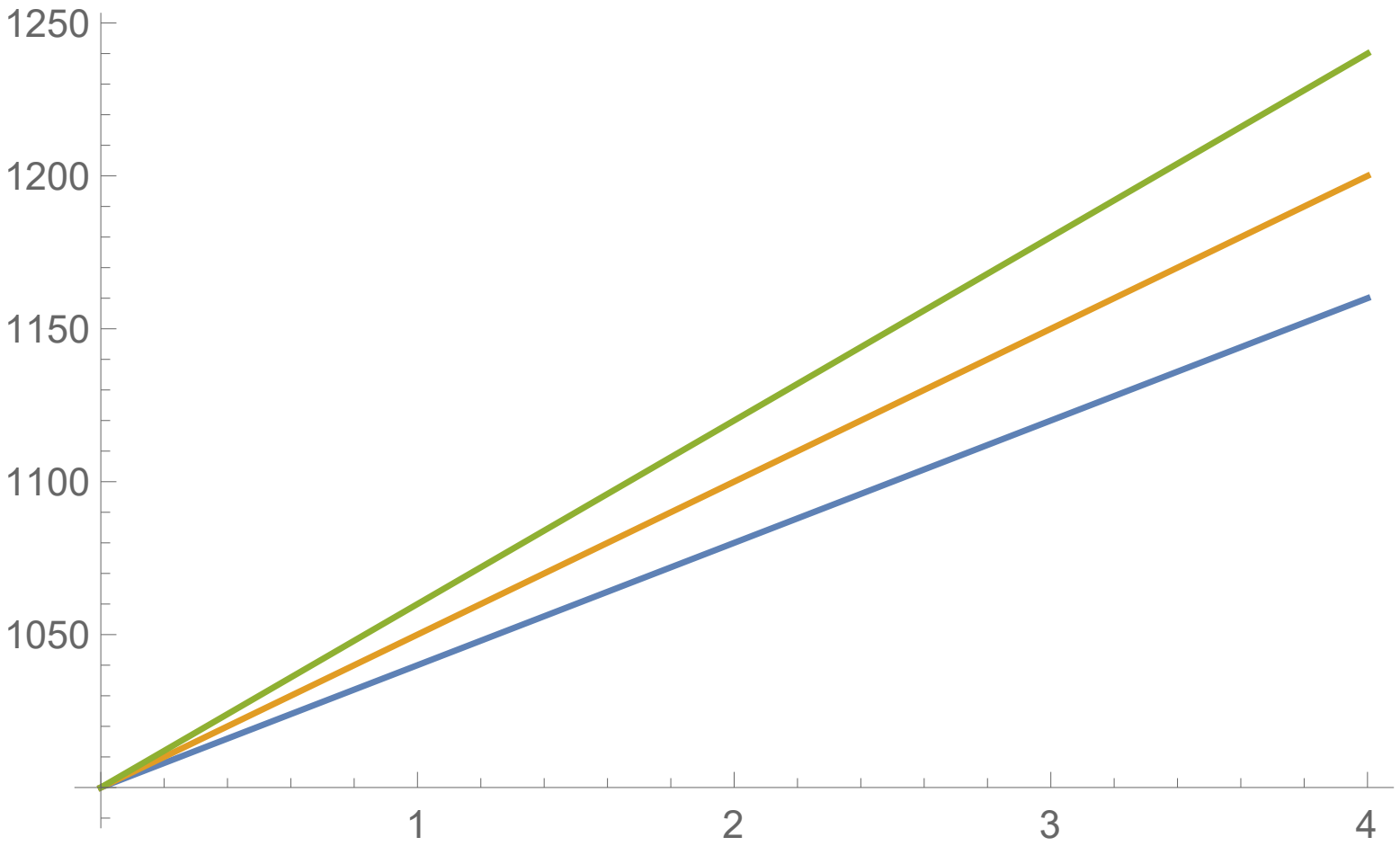
Definition (The Rule of 72)

The Rule of 72 says that an interest rate of $r\%$ will double the value of an investment in approximately $72/r$ years.

- An effective rate of 9% will double the value of the investment in 8 years.
- By how much will the investment (at 9%) grow in 16 years?
- How long will it take for the future value to grow to 8 times the principal? 16 times?

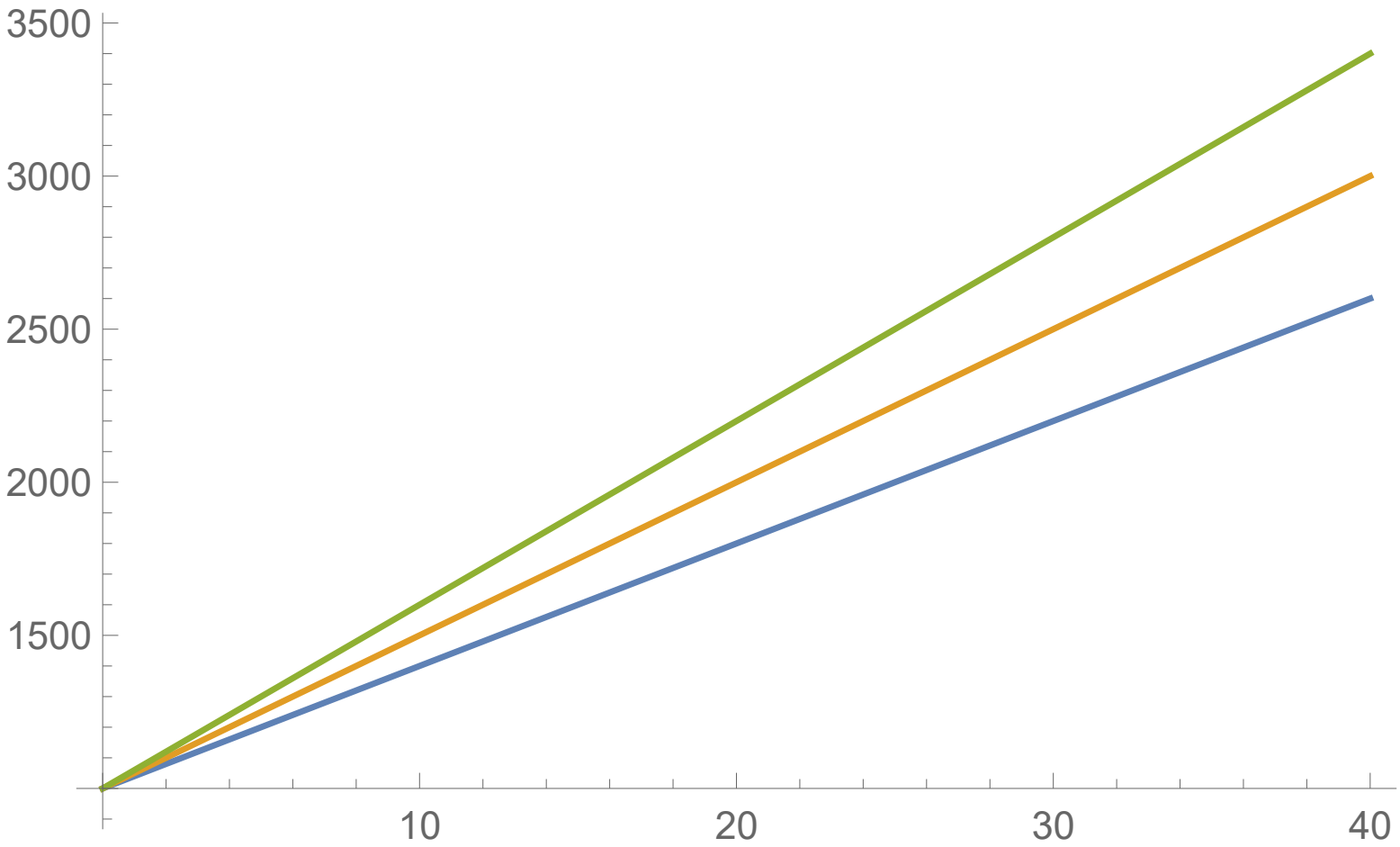
Outline

Simple vs. Compound Interest



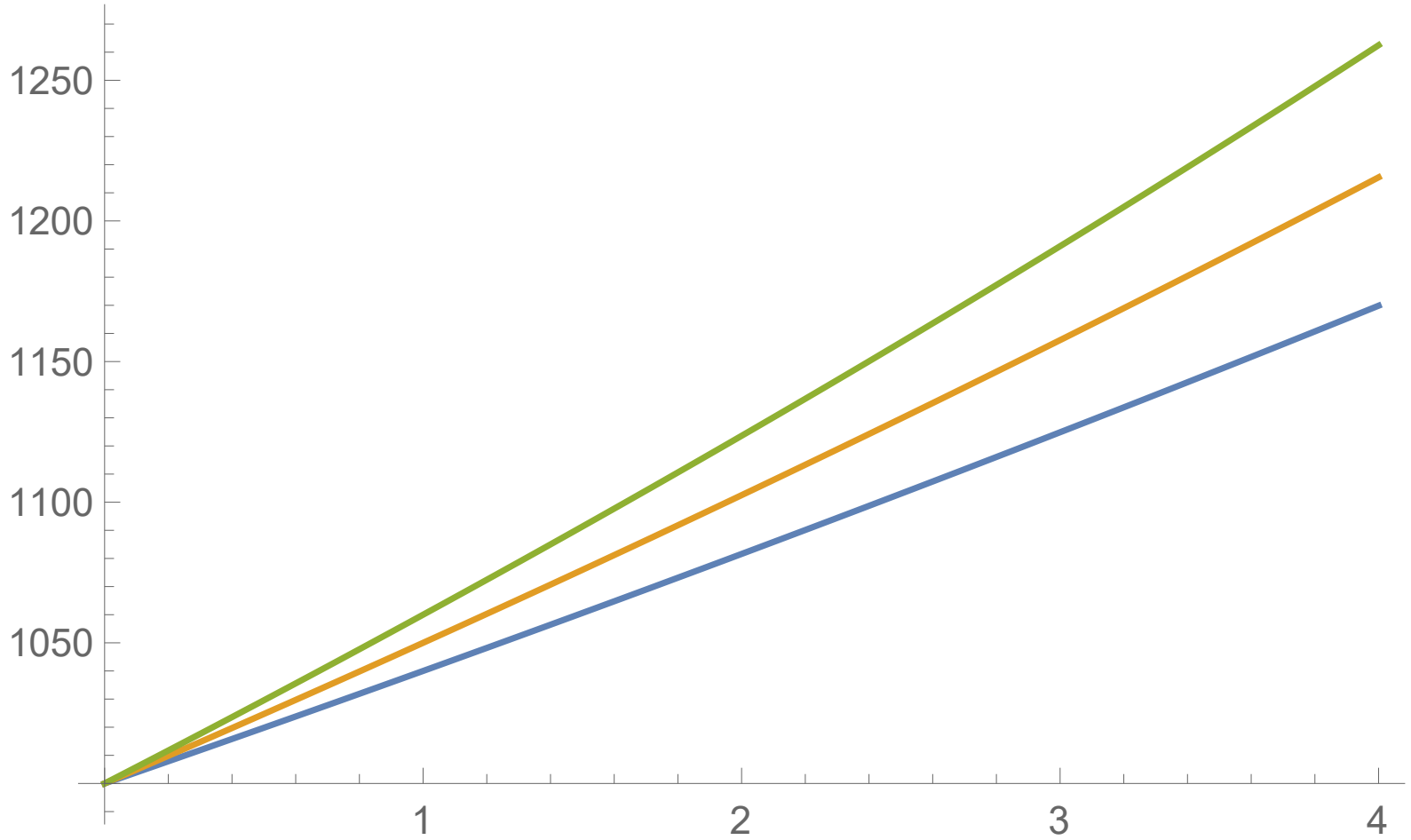
Interest: 4% $\rightarrow$ \$160; 5% $\rightarrow$ \$200; 6% $\rightarrow$ \$240

Simple vs. Compound Interest



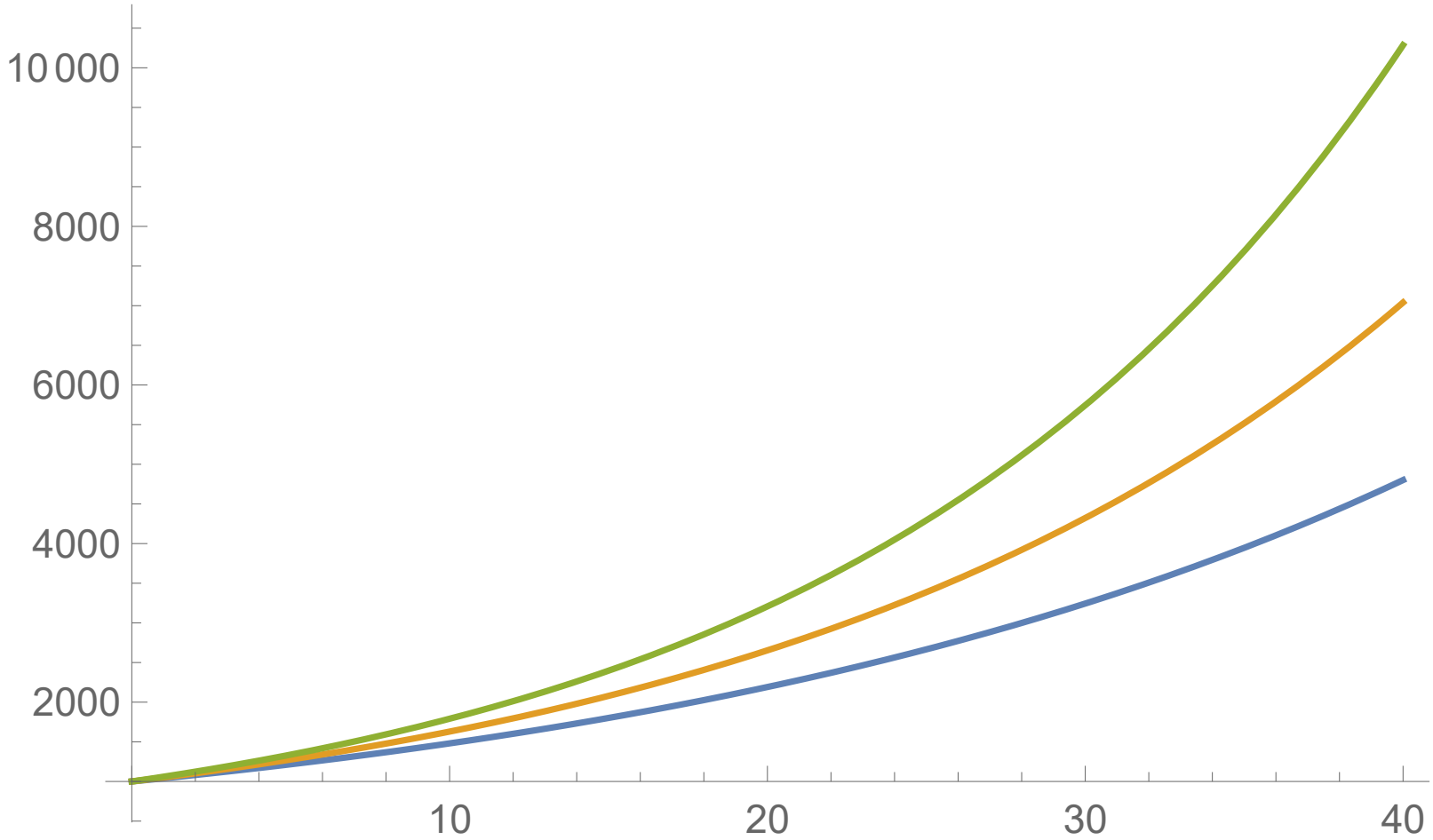
Interest: 4% $\rightarrow$ \$1,600; 5% $\rightarrow$ \$2,000; 6% $\rightarrow$ \$2,400

Simple vs. Compound Interest



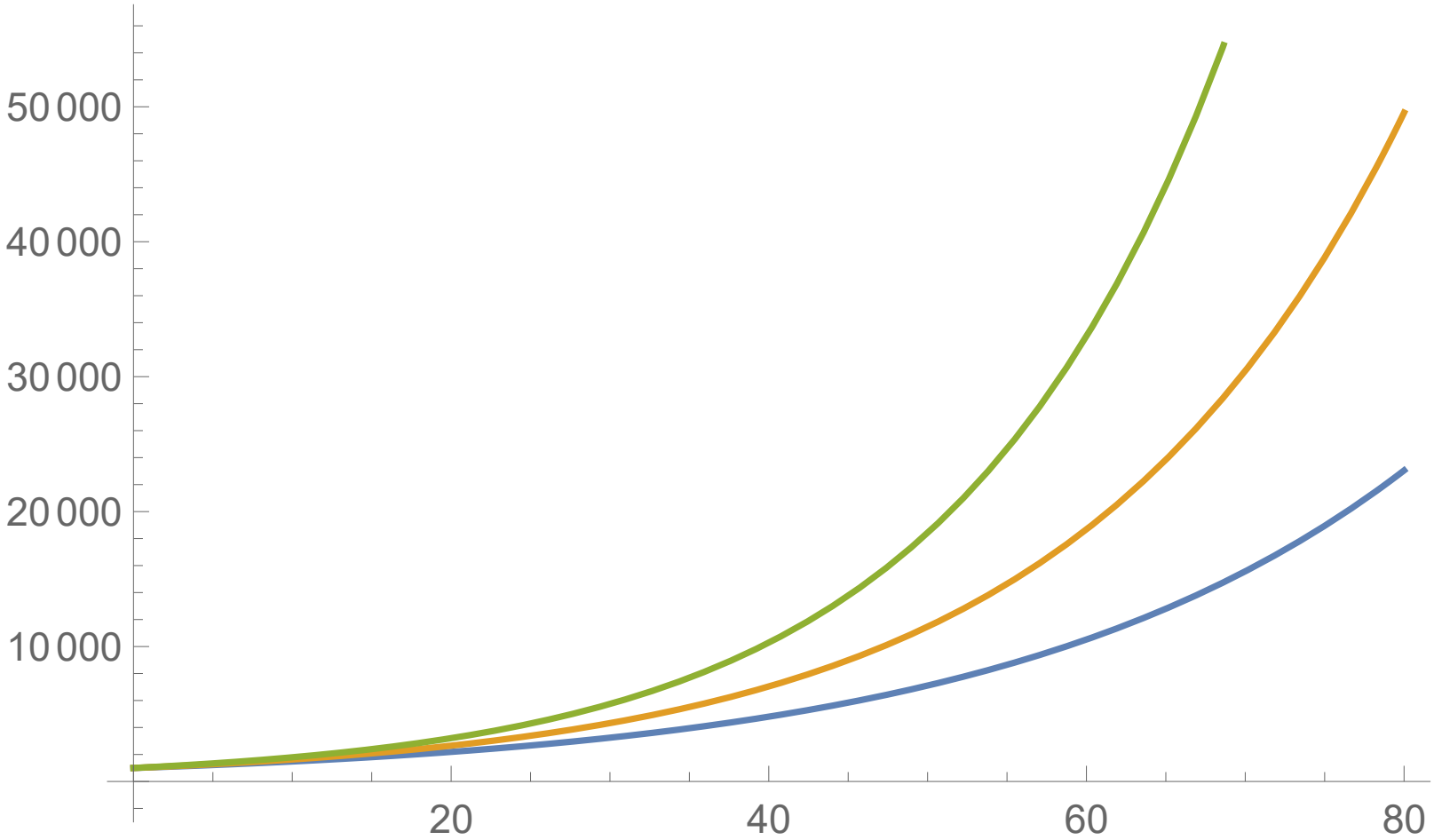
Interest: 4% $\rightarrow$ \$170; 5% $\rightarrow$ \$216; 6% $\rightarrow$ \$262

Simple vs. Compound Interest



Interest: 4% $\rightarrow$ \$3,801; 5% $\rightarrow$ \$6,040; 6% $\rightarrow$ \$9,286

Simple vs. Compound Interest



Interest: 4% $\rightarrow$ \$22,050; 5% $\rightarrow$ \$48,561; 6% $\rightarrow$ \$104796

Outline

The Compound Interest Formula

- If the interest is compounded annually, then

$$F = P(1 + r)^t.$$

- If the interest is compounded k times a year, then

$$F = P\left(1 + \frac{r}{k}\right)^{kt}.$$

Compound Interest

Example (Compound Interest)

- A principal of \$1000 is invested at 4% interest compounded quarterly for 4 years.
- Find the future value.

Compound Interest

Example (Compound Interest)

- A principal of \$1000 is invested at 4% interest compounded quarterly for 4 years.
- Find the future value. ans: \$1,172.58

Compound Interest

Example (Compound Interest)

- A principal of \$1000 is invested at 4% interest compounded quarterly for 4 years.
- Find the future value. ans: \$1,172.58
- Find the future value if it is compounded monthly.

Compound Interest

Example (Compound Interest)

- A principal of \$1000 is invested at 4% interest compounded quarterly for 4 years.
- Find the future value. ans: \$1,172.58
- Find the future value if it is compounded monthly. ans: \$1,173.20

Compound Interest

Example (Compound Interest)

- A principal of \$1000 is invested at 4% interest compounded quarterly for 4 years.
- Find the future value. ans: \$1,172.58
- Find the future value if it is compounded monthly. ans: \$1,173.20
- Find the future value if it is compounded quarterly for 50 years vs. simple monthly for 50 years.

Compound Interest

Example (Compound Interest)

- A principal of \$1000 is invested at 4% interest compounded quarterly for 4 years.
- Find the future value. ans: \$1,172.58
- Find the future value if it is compounded monthly. ans: \$1,173.20
- Find the future value if it is compounded quarterly for 50 years vs. simple monthly for 50 years. ans: \$7,316.02 vs. \$7,364.52

Outline

The Effective Interest Rate

Definition (Effective Interest Rate)

For a given compound interest rate, the effective interest rate r_{eff}, or annual percentage yield (APY), is the interest rate that produces the same yield when compounded annually.

- Given an interest rate r compounded k time a year, the effective rate is given by

$$\text{APY} = \left(1 + \frac{r}{k}\right)^k - 1.$$

The Effective Interest Rate

Definition (Effective Interest Rate)

For a given compound interest rate, the effective interest rate r_{eff}, or annual percentage yield (APY), is the interest rate that produces the same yield when compounded annually.

- Given an interest rate r compounded k time a year, the effective rate is given by

$$\text{APY} = \left(1 + \frac{r}{k}\right)^k - 1.$$

- This is the same as computing the interest earned on \$1.00 after one year.

The Effective Interest Rate

Example (Effective Interest Rate)

- Find the effective interest rate, or APY, of 12% compounded
  - Annually

The Effective Interest Rate

Example (Effective Interest Rate)

- Find the effective interest rate, or APY, of 12% compounded
  - Annually
  - Semiannually

The Effective Interest Rate

Example (Effective Interest Rate)

- Find the effective interest rate, or APY, of 12% compounded
  - Annually
  - Semiannually
  - Quarterly

The Effective Interest Rate

Example (Effective Interest Rate)

- Find the effective interest rate, or APY, of 12% compounded
  - Annually
  - Semiannually
  - Quarterly
  - Monthly

The Effective Interest Rate

Example (Effective Interest Rate)

- Find the effective interest rate, or APY, of 12% compounded
  - Annually
  - Semiannually
  - Quarterly
  - Monthly
  - Daily

The Effective Interest Rate

Example (Effective Interest Rate)

- Find the effective interest rate, or APY, of 12% compounded
  - Annually
  - Semiannually
  - Quarterly
  - Monthly
  - Daily
  - Hourly

The Effective Interest Rate

Example (Effective Interest Rate)

- Find the effective interest rate, or APY, of 12% compounded
  - Annually
  - Semiannually
  - Quarterly
  - Monthly
  - Daily
  - Hourly
  - Minutely

The Effective Interest Rate

Example (Effective Interest Rate)

- Find the effective interest rate, or APY, of 12% compounded
  - Annually
  - Semiannually
  - Quarterly
  - Monthly
  - Daily
  - Hourly
  - Minutely
  - Secondly

The Effective Interest Rate

Example (Effective Interest Rate)

- Find the effective interest rate, or APY, of 12% compounded
  - Annually
  - Semiannually
  - Quarterly
  - Monthly
  - Daily
  - Hourly
  - Minutely
  - Secondly
- Do you notice a trend?

Outline

Certificates of Deposit

Definition (Certificate of Deposit)

A certificate of deposit (CD) is an investment, usually through a bank, for a fixed period of time at a fixed compound interest rate.

- There is typically a penalty for early withdrawal. The penalty is usually equal to 6 months of interest.

Certificates of Deposit

Example (Certificate of Deposit)

- What is the future value of a 5-year CD earning 2% interest, compounded quarterly?

Certificates of Deposit

Example (Certificate of Deposit)

- What is the future value of a 5-year CD earning 2% interest, compounded quarterly?
- Suppose that at the end of one year, the rate on new CDs is 3%. Should the investor cash in the CD earning 1%, pay the penalty of 6 months' interest, and reinvest the difference in a new CD earning 3%?

Certificates of Deposit

Example (Certificate of Deposit)

- What is the future value of a 5-year CD earning 2% interest, compounded quarterly?
- Suppose that at the end of one year, the rate on new CDs is 3%. Should the investor cash in the CD earning 1%, pay the penalty of 6 months' interest, and reinvest the difference in a new CD earning 3%?
- What if the penalty were 12 months' interest?

Outline

Assignment

Assignment

- Chapter 10: Exercises 27, 28, 31, 32, 41, 42, 43, 44, 70.